

Basic mathematical conceptualisation as pragmatically over-determined

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Abstract: How one may articulate inside the notion of conceptualisation both the cognitive and inter-subjective issues of the initial mathematical thinking that emerges from didactic systems? The present work studies critical terms of this articulation. It leads to an understanding of conceptual invariants as pragmatic cognitive elements that simultaneously refer to epistemological and contractual dimensions of mathematical activity inside the instructional institution.

Keywords: conceptualisation, social cognition, development, learning, didactic contract, mathematics education.

1. Introduction

The main finality of primitive conceptual mechanisms is to efficiently perform tasks and not to erect theories. We study the case of mathematical techniques learning and observe, initially at least, that many students tend to face didactic injunctions by developing finalised conceptual products (Pichat & Ricco, in press, b). These latter ones are pragmatic knowledge that is supposed to insure a direct mastery of mathematical techniques that were taught and whose mastery is to be exhibited in order to try to show oneself as a good subject of the didactic institution.

Pragmatic knowledge is constituted of basic conceptual invariants whose design is to maximise performance inside the didactic institution at a minimal cognitive cost. It denotes elementary conceptual products that tend to allow, through minimisation of conceptual labour, the display of the control of mathematical techniques that are the objects of the learning process evaluation (Pichat & Ricco, a). This genuine basic knowledge results from a finite appropriation of the mathematical issues of these techniques. It permits to bypass an extended elaboration of mathematical issues in favour of an immediate conceptualisation of limited objects, properties and relations that are thought to be functional according to what is thought to be the teacher’s expectations. From the integration of two perspectives, one rooted in developmental psychology and one rooted in social developmental psychology, it is tried to understand in a unified way both epistemological and social issues of (initial) pragmatic mathematical conceptualisation.

2. Conceptual knowledge as being pragmatic

Through the perspective developed by Vergnaud (1982), knowledge is understood to be fundamentally pragmatic since it is defined as a competence in a situation. To a large extent, conceptual knowledge does not refer to theory but action. As such, the scheme, which is the invariant organisation of cognitivo-gestural activity corresponding to a class of situations, is a dynamic, structured and functional totality whose main finality is efficiency (Vergnaud, 1998).

Knowledge-in-act, as conceptual invariants, are the core elements of the scheme. This elementary knowledge refers to both (i) categories of thinking that are thought to be functional (concepts-in-act) and (ii) pragmatic propositions that are considered to true and which generate possibilities of action (theorems-in-act).

Concepts-in-act sustain cognitive activity in terms of objects of thinking through which subjects capture and integrate the indispensable information that is just necessary for action to be efficient (Vergnaud, 1999). These vectors of thinking organise information processing since they are the cognitive filters throughout which the subject shall cut into and perceive, in a specific way, reality according to the singular goals of his/her finalised activity (Pastré, 1997). Theorems-in-act also structure cognitive activity but in terms of instanced properties, relations and conditions that practically allow and found the functional character of action (Vergnaud, 1999). They permit this functional quality since they are the cognitive elements that direct the elaboration of rules of action (procedures); to that extent, rules of action are potential operationalised consequences of theorems-in-act with which these rules hold, *de facto*, a relative morphemic continuity. But theorems-in-act also found the efficacy of activity by basing it on the practical theories these propositions constitute, these propositions being elaborated from the combination of the properties of the reality on which subjects tend to efficiently perform.

Conceptual representation is a pragmatic economy of thinking that allow the subject to focus his/her attention on the sole elements that are, correctly or not, estimated to be functional with regard to action (Vergnaud, 1999; Pastré, 1997). This point does not mean that conceptual knowledge may not develop into more elaborated and theoretical forms. Not at all. But the fact is that initially, the epistemic nature of knowledge is limited to a major pragmatic aspect.

3. Cognitive activity as being influenced by contractual inferences

Social psychology of development, notably from the work of L.S. Vygotski (1962), stresses the intrinsic social nature of thinking and cognitive mechanisms that empower it. In a didactic situation of task performing, this approach emphasises the interactions between both the social and cognitive dimensions of thinking by outlining the interpretative activity of the subject who aims at identifying the specific “implicit contract” that is to be satisfied *hic et nunc* (Schubauer-Leoni & Grossen, 1993). This contract is not a real contract, as it is not the result of a formalised agreement between students and teachers. Although then the expression “didactic contract” may be misleading, it looks like as if an implicit contract structures and organises any situation of communication (Schubauer-Leoni & Perret-Clermont, 1997). For instance, if a student has not performed what he/she was supposed to do, such as applying a mathematical technique improperly or incorrectly, or in a non-relevant way, one may say he/she breaks the contract (Brousseau, 1996). This is due to the fact that when the teacher proposes a task to the student, he/she expects a specific conduct of the latter. When students

do not behave in the anticipated way with regard to a given mathematical task, the teacher makes them understand, explicitly or implicitly, that they did not perform what was (implicitly) expected from them.

Subject's cognitive activity is based on his/her contractual definition of the situation that is to be faced. Through this contextual definition, subjects try to determine the status and the role they have to respect, the rights and duties that are incumbent upon them, the kind of proofs they have to exhibit in order to manifest a relationship to knowledge that match the official expected relationship. In one word, the contract they have to fulfil *hic et nunc*. During a problem solving situation, pupils attempt to infer "what is to be answered", what is expected, thanks to the decoding of an implicit contract they (correctly or not) infer. To that extent, the notion of "didactic contract" (Brousseau, 1996) refers notably to the (specifically linked to definite knowledge) set of behaviours that are supposed to be expected from the students by the teacher.

Cognition in a didactic situation is therefore fundamentally inter-subjective and cannot be reduced to intrinsic characteristics of the student himself, by neglecting the specific and singular way this subject represents the contractual issues of the situation (Schubauer-Leoni & Perret-Clermont, 1997). The cognitive activity that is inferred by the researcher may not be simply limited to "an internal logic of the solver conduct" (Schubauer-Leoni & Ntamakiliro, 1994, p.92) and not consider the institutions in front of which students effort to show themselves as good subjects. Therefore, it is necessary to both regard "the subject/environment indissociable relationships by understanding cognitive activity inside a inter-individual, cultural and institutional context" (idem, p.88) and set interaction as the minimal unit of cognitive analysis (idem).

4. How cognition and inter-subjectivity may be articulate?

Vergnaud outlines that the theory of schemes he develops mainly refers to the intra-individual conceptual functioning of subjects and that this theorisation may not always be able, on its own, to give account of the specific and singular contractual processes mobilised by these subjects when they have to face the social interactions that are proper to the didactic system. Therefore, the understanding of the effective conceptual working of the schemes that are elaborated by students is to be deepen. But then, "different questions are raised [...]: does this imply schemes having a social nature?, cognitive nature? At which level of rationality are they situated? To which theoretical universes do they have to be referred to?" (Schubauer-Leoni, 1996, p.188). It is possible to postulate the existence of specifically social schemes that would co-ordinate with properly cognitive schemes according to the perspective developed by J. Brun; fruitful outcomes are to be expected from this genuine orientation. However, our contribution to this questioning is to suggest an alternative direction of investigation: the immediate integration of the social dimension (via the notion of didactic contract) into the very mechanisms of conceptualisation and into the cognitive schemes themselves. In fact, it appears to be possible to develop the notion of conceptualisation toward this orientation since Vergnaud stresses the multidimensional and heterogeneous nature of this powerful instrument of psychological analysis (Vergnaud, 1999).

The articulation, inside subject's conceptualisations themselves, of both the cognitive (intra-individual) and social (inter-individual) dimensions of mathematical thinking, inside the didactic institution, is the major issue of an investigation we are conducting at the moment.

This presentation of this research shall be limited, in the present communication, to the following theoretical double questioning : how the notions of conceptualisation and didactic contract may be articulate? What may be the nature of this potential articulation?

5. Some necessary conditions to the articulation of cognitive and contractual processes

5.1 Questions about the level of articulation of cognitive and contractual processes

The first matter we shall argue refers to the sensitivity and to the analytical “grain” of the articulation of cognitive and social processes of mathematical thinking inside the didactic system. A strong position developed by Vergnaud (1982) is that it is not possible to understand, in a satisfying manner, cognitive functioning without considering the singular objects on which this functioning operates. This implies that the specificity of the objects, cognition applies to, induces a precise form of cognitive processing. To that extent, fine epistemological analyses are required so as to investigate the distinct conceptualisation of each knowledge that is the object of students’ learning. Of course, in a scientific perspective, generality of conceptual processes is to be aimed at; but the stable properties of this generality seem only to be extractable from the invariance of local epistemological phenomena of knowledge acquisition (Vergnaud, 1999).

If the analysis of the conceptualisation of finite objects of knowledge tends to be articulated to the inter-subjective and contractual mechanisms that penetrate this conceptualisation, this cannot be realised without considering what accurately occurs at the local epistemological level of this interaction. Therefore, it is necessary to focus on precise didactic contracts that refer to authentically knowledge-based behaviours the student thinks to be expected from him/her by the teacher. To bypass these inferences would conduct to neglect the genuine knowledge-based “micro-contracts” that product local significant effects on the conceptualisation of the knowledge that is studied inside the didactic system (Pichat & Ricco, in press, b). Therefore, one first positioning of the articulation of cognitive and social processes is proposed. First postulate: if it is intended at understanding the genuine issues of the appropriation of the objects of learning, the articulation of the cognitive and social mechanisms must be worked at the level of the conceptualisation of finite mathematical objects that are worked by precise inferred “micro-contracts”.

5.1 Questions about the subjective meaning of the articulation of cognitive and contractual processes

What is the finality of students’ conceptual activity inside the didactic system? One who effectively knows school classes is aware that students in mathematics do not only care, *a priori*, about mathematics as such. Students do not indeed, *a priori*, only try to perform mathematical tasks *per se*: they attempt to efficiently face implicit contractual injunctions that concern mathematics. Therefore, students’ activity is not similar, even in a very simplified way, to the activity of the professional mathematician. This is all the more true since, usually, students try to maximise performance despite a minimisation of the conceptual cost that is necessary for this. To that extent, subjects’ motives are pragmatic (1997). Students conceptual activity is set at another level: the level of efficiently managing didactic tasks calling for the exhibition of their ability to correctly use mathematical

techniques (that have been institutionalised in the class) accordingly to implicit contracts that have to be inferred in each situation of task processing.

The mathematical techniques that are proposed to students during education therefore subjectively acquire a contractual value since these are the objects through which a judgement of conformity will be performed by the didactic institution (Chevallard, 1996). The mathematical knowledge that is studied in the class subjectively possesses for the students a dimension of evaluation of their institutional submission: every time that are presented a new mathematical technique, students become aware that they will somehow have to display what is (supposed to be) expected from them concerning this technique and will have to try to satisfy the local contract that refers to this knowledge. Our second postulate is thus the following: the subjective finality of the conceptual activity that is performed by students in didactic systems is essentially pragmatic and not epistemic since this activity aims at exhibiting behaviours that match what students think to be the didactic contract.

5.3 *Questions about the type of articulation of cognitive and contractual processes*

Many investigations of knowledge development do not consider its social dimension. Other ones link, in various ways, these cognitive and inter-subjective dimensions. But most of these works juxtapose instead of integrating the cognitive and inter-subjective issues of knowledge development. The recurrent postulate of these perspectives is the following: social mechanisms produce effects, from the outside, onto the construction of knowledge. It seems that it is seldom tried to understand as intrinsically indissociable conceptual development and inter-subjectivity.

Only the Russian perspective, with notably Vygotski (1962), appears to be effectively supporting the matter of the difficulty to think conceptual activity as implying functions that would not fundamentally be inter-psychical. According to this perspective, knowledge is neither influenced nor modulated by social phenomena: knowledge is social in essence. In other words, conceptualisation is understood as being the result of an inter-individual construction; it is the product of a social mediation and develops through an internalisation of what is initially shared in the space of inter-subjective relations. This strong and genuine position is clearly explicated by Vygotski in his analysis of the inter-related development of “daily-life” and “scientific” concepts. And it is still the same idea that is expressed through the notion of proximal zone of development. This conducts to our third postulate: the cognitivo-social duality of thinking must be inscribed in an integrative way inside the conceptualisations themselves that sustains this thinking.

6. A proposal: pragmatic conceptual knowledge as simultaneously epistemological and contractual

6.1 *Over-determined conceptualisations*

In order to elaborate a theoretical framework working the three issues that were pointed out in the past sections, the following hypothesis, which is judged to be central in the analysis of students’ cognitive activity inside the didactic system, is proposed: it is difficult to artificially dissociate mathematical knowledge and knowledge about the didactic system in which this knowledge evolves (Pichat & Ricco, in press, b). In other words, we postulate that

the conceptual knowledge which is elaborated by students would possess, at once, a double cognitive status: a mathematical status and a didactic status. To that extent, this pragmatic knowledge would be a “mathematico-didactic” one (Pichat & Ricco, in press, a). This knowledge would be over-determined and condense several senses, distinct axes of rationality: those of the mathematical objects that are involved and those of the didactic system concerning these objects.

Therefore, this conceptual knowledge would not only allow to identify the critical elements that are effective for action in a situation but as well, given performed actions are to match inferred contractual expectations, to identify *de facto* the determining elements that permit to base action on the actions that are thought to be expected. Thus, pragmatic knowledge would let operate on an environment that would not be purely mathematical but mathematico-didactic since cognitive activity is oriented toward teachers expectations. According to this perspective, to know how signifies to know how to fulfil expectations: subject acts on a reality that jointly possesses mathematical and didactic properties.

It would then seem to be necessary to understand a supplementary functionality of conceptualisation: a contractual one. Conceptualisation would sustain extraction of objects, properties and relations that are endowed with a double pragmatic feature since these three conceptual categories allow both the specification of the goal that is to be reached and the implementation of the schemes that permit the manifestation of institutional techniques (Pichat & Ricco, in press, a). That is, concepts-in-act would simultaneously possess an epistemological value, such as it is described by the theory of Vergnaud (1999, 1998, 1982), and also a contractual value as they allow cognition to identify and extract the objects activity is to operate on in order display what is thought to be expected by the teacher. Similarly, theorems-in-act would simultaneously possess an epistemological value and a contractual one as they determine how it is to be operated on the mentioned extracted objects in order to exhibit what is locally expected (i.e. to satisfy the involved inferred micro-contract).

6.2 *Observable phenomena that give account for the postulated cognitive duality*

From our theoretical perspective, we predict the experimental observation of two singular phenomena that would be characteristic of the pragmatic duality of knowledge such as it was presented. The first phenomenon refers to the selection, in a situation, of a scheme to be mobilised *hic et nunc*. The second one, which is posterior to the selection of a scheme, is concerned with the knowledge that will manage the scheme.

We identify the first phenomenon as an affordance of a contractual nature. By analogy with the notion developed by Gibson (1979), the contractual affordance would refer to the fact that a given specific configuration of a task that is proposed to students incite them, through a simultaneously epistemological and contractual pragmatic conceptualisation of this task, to preferentially mobilise a particular scheme that is relevant for exhibiting the mathematical behaviour that is called by the didactic contract which is thought to be decoded. For instance, when facing the algebraic form $(x+2)^2$ in a given task, a secondary school student may get to think that the teacher expects him/her to exhibit the ability to recognise and utilise the algebraic property “ $(a+b)^2=a^2+b^2+2ab$ ”. So this student will unconditionally do so and this no matter whenever it is relevant or not given the task that is effectively to be performed (for instance that is not relevant if the task is to solve linear equations such as $(x+2)^2=9$).

The second phenomenon is identified as a focusing of a contractual nature. By analogy with the notion that was proposed by Piaget, the notion of contractual focusing tries to give account for the fact that a scheme will be organised around a partial pragmatic conceptualisation that allow students to operate accordingly to a way they (often wrongly) think to be conformant with the specific involved contract. In other words, contractual focusing implies a conceptual knowledge that structures activity by getting it to focus on a sole property (having both an epistemological and contractual value) that is thought to be efficient for manifesting the behaviour that is expected by the teacher. For instance, we begin to observe a case concerning the calculation of absolute values (let us say $|2-\pi|$): numerous students systematically transform the negative signs into positive one (" $|2-\pi|=2+\pi$ ") accordingly to their (incorrectly assimilated) teacher explanation which is "to apply an absolute value to an expression is to make it positive".

7. Conclusion

Mathematical conceptualisation implies epistemological pragmatic issues : it relies on knowledge-based categories of thought that are considered to be useful and on practical propositions that are judged to be functionally true (Vergnaud, 1999). But it also involves pragmatic inter-subjective processes and depends on inferences about what is expected by the teacher, through the decoding of implicit didactic contracts (Schubauer-Leoni & Perret-Clermont, 1997). We propose to understand this cognitivo-social pragmatic duality of mathematical thinking as inserted into mathematical conceptual knowledge itself.

From a theoretical point of view, efforts have to be conducted in order to deepen the understanding of (initial) mathematical cognition as simultaneously oriented toward the task itself (epistemological dimension) and toward the decoding of expected patterns of behaviour (contractual dimension). One fruitful way to better understand learning mechanisms and cognitive functioning, may lie in expanding the articulation of conceptualisation such as it is defined by the work of Vergnaud (1999, 1998, 1982), with the works from the current socio-constructivistic researches (let us mention M.L. Schubauer-Leoni but also M. Brossard and J. Brun). But the point is to ensure this articulation will form an effective integration and not just an additive juxtaposition.

Many issues are raised by the consideration of the postulated conceptual duality of knowledge. One of them is to understand how pragmatic knowledge may give a genuine form to mathematical reasoning by orienting it toward one specific direction in the "task space" possibilities (i.e. selecting one specific cognitive scheme among numerous other possible ones). An other interesting stake is to understand why some students never overcome the utilisation of this basic kind of knowledge whereas other ones may internalise and re-elaborate it so as to go beyond this initial cognitive functioning (the Vygotskian notion of internal proximal zone of development such as presented and worked by M. Brossard is enlightening here). A third point is to study how the didactic system itself may paradoxically favour the emergence of this pragmatic knowledge by letting it to be functional in many situations and maybe even by inducing (weak) students to construct it.

8. References

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