Lynda R. Wiest
University of Nevada, Reno
October 2003

Comprehension of Mathematical Text

Language proficiency and mathematics proficiency appear to be linked, such that lower language proficiency tends to translate into poorer mathematics performance (Cocking & Mestre, 1988; MacGregor & Price, 1999). This not only points to the role of literacy skills in learning other content areas, but it also has important implications for constructing, selecting, and teaching mathematical word problems.

In our continued efforts to improve word problems to stimulate student thinking, without inadvertently hindering performance, we must consider readability issues. Readability includes all factors related to reading and comprehending written text. As Thomas (1997) points out:

> It is unlikely that standard readability formulas will ever be able to offer real help in coming to understand what it is about mathematics embedded in text that makes it difficult to read. A far more important task is identifying areas of difficulty. One is context...another is linguistic difficulty. (p. 40).

Despite its importance, however, "Research in reading and mathematics continues to attract little attention" (Thomas, 1997, p. 39).

In this article, I describe factors that influence text readability, in particular, literacy factors that influence comprehension of mathematical text.

Assessing Readability of Text

Readability is a critically important issue to consider in constructing word problems and analyzing student performance in solving them. However, it does not seem to be gauged very accurately by the quantitative measures used by many standard methods, such as that of the Fry graph, which bases scores on an average number of syllables per 100 words (cf. Agnihotri & Khanna, 1992). A speaker in any language can easily imagine that two passages with the same average number of syllables per sentence can differ vastly in comprehensibility. Word and sentence length are often key considerations in standard readability formulas. However, small words—prepositions, for example—can be particularly problematic for readers (MacGregor, 1993; Rastall, 1994). For one thing, prepositions may have multiple meanings and thus only acquire meaning in context (e.g., *in* the box versus *on* the box), making them quite difficult to translate among languages (Smith, 1994). Readability scores can also vary by the particular formula employed. For example, the following word problem scores 5.2 on the Flesch-Kincaid grade-level scale and 7.6 on the Gunning Fog Index:

> Of the 157 music cassettes that their older sister had collected, Adam and Kelly counted 137 songs that they like the best. Of the songs that they like the best, 48 are rock songs, 47 are country songs, and the rest are rap songs. How many songs of those Adam and Kelly like best are rap songs?

The present emphasis on a more contextual, balanced—besides phonetic—approach to literacy makes such atomizing techniques suspect. Many researchers question or dispute use of standard readability measures (cf. Shelby, 1992), favoring approaches that "focus on syntax, conceptual difficulty, and organisation" (Agnihotri & Khanna, 1992, p. 282). The limitations of determining the readability of mathematical text are particularly great (Paul, Nibbelink, & Hoover, 1986; Shuard & Rothery, 1984). Rye (1985) says readability formulas should be used for crude prediction, such as ordering the relative difficulty of texts, and they should be used in conjunction with other factors, such as teachers' estimates.

Readability scores determined by standard formulas do not appear to capture comprehensibility of word problems. Paul et al.'s (1986) research involving more than 1000 third- through sixth-grade students showed that this type of readability bore no relationship to students' ability to solve the problems. Other researchers have also found no correlation between standard readability scores and problem-solving performance (Hembree, 1992; Wiest, 1996/1997).

Most standard readability measures cannot account for concept load in a verbal piece, children's background knowledge and experiences related to content, or children's interests (C. Nelson, personal communication, December 14, 1995; A. Newman, personal communication, June 3, 1996). After reviewing literature on readability measures and assessing Hindi text with several standard English readability formulas, Agnihotri and Khanna (1992) concluded:

> Readability of a text, whether in Hindi or English, depends not only on word and sentence length but also on its linguistic and conceptual organisation. It also depends on the assumptions a writer makes about the prior knowledge of the reader and the writer's success in facilitating the reader's access to the text's content and textual schemata. (p. 288)

Allen (1985) found that students better understood text that they themselves generated than text created by adults, even though their text had longer sentences and measured at a higher readability level according to the Spache formula. She speculates that not only greater familiarity of the text but also the *longer sentences* contributed to the findings, the latter serving to fill information gaps often found in adult-written material.

Allen states, "We are learning that no text is easy or difficult in and of itself. It is the interplay between reader and text that determines comprehension" (p. 1).

It seems reasonable to expect that readability in the broader sense influences student problem-solving performance. Familiarity with the story content itself can influence understanding of the text. In my research (Wiest, 1996/1997), for example, many upper-elementary students did not know the meaning of "write-in vote," which appeared in one problem. Although I had explained the term before they encountered it, many students used it inappropriately and thus solved the problem incorrectly. Students also struggled with saying easy-to-pronounce but unfamiliar words, particularly names, such as Kline, Shonda, and Chen. In a written response to the problems the students had solved, a sixth-grade girl suggested, "Use easy to pronoce [*sic*] names." Complex wording and obscure concepts also frustrated students. One sixth-grader said she dislikes complicated wording, and another said she would like to see word problems use "kids' language" because she can understand it better.

One factor considered important in reading is interest in the text, which—along with background knowledge—can predict readability much more strongly than can standard formulas (Anderson, Higgins, & Wurster, 1985; Anderson & Davison, 1986). In a study of free-reading selections, Anderson et al. (1985) found that fourth and sixth graders of all reading-achievement levels chose books of similar readability, thus making the text topics more influential than readability. Beck, McKeown, Omanson, and Pople (1984) revised basal-reader stories to improve their coherence, for example, by filling potential knowledge gaps, and found that both lower- and upper-ability students showed improved comprehension even though standard readability scores indicated increased difficulty.

Reading for Different Purposes

People use different strategies to read different materials, for example, a story, a stock market report, and a mathematical word problem. Student interest in particular story content in a piece of literature might not carry over to the same story content used in a mathematical word problem, or vice versa. One survey of sixth through eighth graders' interests found that such a correlation between the two forms does not always exist (Theule-Lubienski, Burgis, & Keiser, 1995).

Reader response to written material is determined by various personal, textual, and contextual factors (Galda, 1990). One factor is a reader's expectation for reading. The form and style of reading material, as well as the purpose for reading it, impact how individuals read the material, how they interact with or respond to it, and what they take from the experience. For example, Galda (1990) says reading literary text is a particular kind of language act that positions the reader as a spectator of someone else's experience. Unlike participants in the real world, he says, readers strike a more detached pose in which they are more likely to engage in fuller exploration of feelings and more evaluative thinking (Galda, 1990; see also Wade, Schraw, Buxton, & Hayes, 1993).

Based on their perceptions of texts, people invest differential amounts of mental effort in learning from different sources, which in turn strongly impacts learning in terms of depth of processing (Salomon, 1983). For example, a structurally familiar text may cause a reader to make certain assumptions that lead to less mindful processing and thus fail to attend to particulars or to develop fuller mental elaborations. Stimuli that are more demanding, difficult, or novel tend to evoke more invested effort, and thus deeper processing, than do simple stimuli. Fantasy contexts, for example, require higher comprehension skills (Galda, 1990; Norton, 1991). (Context is the verbal or “storied” aspect of a problem statement.)

Individuals solving word problems analyze them in a specialized way that results in building certain kinds of task- or goal-specific mental representations that are most conducive to successful problem solution (Kintsch & Greeno, 1985). In mathematics, problem context may influence a problem solver's initial impression of a problem (Chipman, 1988). Some contexts—sports averages, for example—may evoke problem categorizations based on previous school experiences and thus particularized, and perhaps highly practiced, routinized solution responses. Cobb (1986) says, "The act of formulating a goal immediately delimits possible actions; the goal, as an expression of beliefs, embodies implicit anticipations and expectations about how a situation will unfold" (p. 4).

One result of being schooled in word problems may be to pay only superficial attention to verbal text. Students may call upon learned strategies deemed to be mathematics-appropriate, such as finding key words (e.g., "in all" or "how much more") to signal which operation to apply to the numbers in a problem. Evidence shows that students often guess which solution process to use based on the numbers in the problem, such as how many numbers there are, or the absolute size of the numbers and their size relative to each other (Garofalo, 1992; Sowder, 1989). In these situations, students often do not attend to understanding problems, to building holistic problem representations, or to judging reasonableness of their answers, but rather to skimming for facts to apply to algorithms. Unfortunately, these inappropriate strategies sometimes, or often, work based on the typical format of word problems. Moreover, "Many mathematics problems require students to suspend reality and ignore their common sense in order to get a correct answer" (Boaler, 1994, p. 554). As an example, students figuring costs for purchased items are seldom asked to consider the addition of sales tax, unless the lesson is about percents. Problem solvers usually accept these limitations of word problems because they see the problems as school mathematics rather than real-world forms.

Renninger (1992) says, "The student is not only engaging in a task when he or she works on a passage or a word problem. He or she is confronting the structure of a domain and a canon for its instruction" (p. 385). Fifth- and sixth-grade teachers in her study reported that they emphasized meaning of the text more in reading but focused on the mathematics content in word problems. Therefore, she says, it is not surprising that interest is a more influential factor in reading—which emphasizes the verbal aspect of the text—than it is in mathematics. Bilsky, Blachman, Chi, Mui, and Winter (1986) say, "It is readily apparent that comprehending verbal math problems involves processes different from those involved in comprehending other types of discourse, such as stories" (p. 109). In a study involving fourth- and fifth-grade general education students and mildly mentally disabled adolescents, they determined that students use different inferencing in processing story text compared with the verbal context of a mathematical word problem. Students attended more to story events in the former case and to quantitative information in the latter, mirroring teachers' instruction in the Renninger study. The overall results of their two experiments led Bilsky et al. to conclude, "In view of the apparent sensitivity of children and [mildly mentally disabled] individuals to the effects of context, further exploration of the potential of using context to facilitate comprehension is warranted" (p. 125).

Kintsch (1986) says some text is read for its own sake, whereas other text is a medium for other purposes. He says the linguistic arrangement of the text affects comprehension, because some kinds of language structures are more familiar than others or better fit our expectations for and experience with particular types of text. Word problems, Brown, Collins, and Duguid (1989) say, "are generally encoded in a syntax and diction that is common only to other math problems" (p. 34).

Experienced problem solvers, according to De Corte, Verschaffel, and De Win (1985), can fill in gaps and comprehend ambiguities that less experienced problem solvers have not yet learned to do. They accept certain "between-the-lines" information as "given" in school-form word problems. De Corte et al. say competent problem solvers have well developed semantic schemata for these types of problems, and they solve them conceptually ("top down"). Less able problem solvers depend more on the text, or a "bottom-up" approach. This may be why younger students in Silver, Shapiro, and Deutsch's (1993) research were more concerned about mathematical formalism, such as the form of their answers, in solving and answering word problems than were older students, who paid greater attention to interpreting their answers. All students, however, were more concerned about form than reasonableness of answer, lending additional credence to contentions that word problems are artificial and that students believe in a particularistic manner of solving and answering them.

Rewording problems in ways that make semantic relations more obvious has been found to aid comprehension and solution processes (Davis-Dorsey, Ross, & Morrison, 1991; De Corte et al., 1985). However, Davis-Dorsey et al. (1991) say rewording word problems to facilitate comprehension may be beneficial to younger students who have "less developed schemata for standard textbook problems" (p. 66), whereas for experienced word-problem solvers it could have "the disadvantage of converting standard problem prototypes into less recognizable variations" (p. 67).

In short, it seems that mathematical word problems have their own culture of familiarity, expectation, and response (in terms of probable solution process) into which we immerse and enculturate students. Standard word-problem form may produce in problem solvers a mental set that differs from that used for reading stories or other types of text. This further complicates both the role of context in word problems and how problem context might be constructed in a manner most advantageous to student learning. Lave (1993) summarizes the matter of school-form word problems:

> There is a discourse of word problems—a set of things everyone knows how to say about word problems or that can be expressed in 'word-problemese', issues and questions that come up when people begin to talk about them; and things that are not or cannot be said within this framework.... The problems themselves are stylized representations of hypothetical experiences—not slices of everyday existence. If you ask children to make up problems about everyday math they will not make up problems about their experienced lives, they will invent examples of the genre; they too know what a word problem is. (p. 77)

Although we might fault instructional methods with teaching superficial approaches to solving word problems,[1] while acknowledging the contribution of standard word-problem formats, reading purpose also provides the reader with a lens through which to view a text. MacGregor (1990) points out that a reader of a word problem must search for key information and the relation of important parts to each other. Further, she says, use of prepositions may differ from the way we use them in everyday language.

MacGregor contends that the mathematics curriculum must include teaching students how to read mathematical text, a belief numerous other researchers and educators share (e.g., Khisty, 1995). Armbruster (1986) says textbooks in different content areas should be designed to facilitate access to or construction of schemata appropriate to reading their particular textual forms.

Mathematics-Specific Language Difficulties

Word problems are laced with language that differs from everyday usage and which is thereby potentially difficult for problem solvers (e.g., Zevenbergen, 2000). Mathematics-specific language, such as *numerator*, *hypotenuse*, *minuend*, and *exponent*, must be acquired. Moreover, some common words, such as *table*, *product*, *rational*, and *odd*, carry different meanings in mathematics than in daily language. Even within mathematics, some words have different meanings in space versus number contexts (e.g., *base* and *square*).

Of course, symbolic language is another area of comprehension to master in mathematics. Students must learn symbols for the operations, relational symbols (e.g., > and <), the meaning of parentheses and brackets, and so forth. Moreover, use of symbols may differ among countries. Consider, for example, the fact that some countries use a comma where the United States uses a decimal to separate whole numbers from fractional parts.

Prepositions typically are conceptually challenging, as noted earlier, and they carry important and sometimes confusing functions in mathematics. These small but power-packed words appear in such phrases as *three-fourths of sixteen pizzas* and *reduce by 5 inches*. The same preposition can signify different actions, as in the expressions *3*

---

[1]Silver (1987), for example, notes that <u>after</u> children have received mathematics instruction, many tend to solve

multiplied by 10 versus *3 increased by 10* (Dale & Cuevas, 1992). Dale and Cuevas (1992) note that “prepositions in general and the relationships they indicate are critical lexical items in the mathematics register that can cause a great deal of confusion” (p. 333). Word order, such as *subtract x from y*, can also be perplexing. So, too, can different ways of saying the same expression, such as *24 divided by 8* and *8 divided into 24*. Logical reasoning carries its own mathematics language (e.g., *therefore*, *if-then*) to which students must become accustomed.

Given these mathematics-related language challenges for the native speaker of English, one can only imagine what this means for English as a Second Language (ESL) students. Moreover, other personal factors impact ESL students’ school mathematics experiences. Cultural background, for example, influences a person's mental structures and frames the way she or he views the world, including the discipline of mathematics. This can affect the way individuals group and categorize things, their manner of "logical" thinking, and so forth.

Even if an ESL student speaks English with reasonably good proficiency, another language issue is that mathematical expressions do not always translate directly into other languages, such as use of the phrase "two times less" in Australian vernacular or "twice shorter than" in Polish (MacGregor, 1993). Languages differ in their manner of expressing mathematical concepts, such that the mathematics register (type of language used in a specialized area) does not translate among them readily (Khisty, 1995). MacGregor (1993) concludes, "Mathematics teachers are beginning to realise that every maths lesson should be also a language lesson, providing opportunities for all students to develop their English language skills" (p. 31).

---

problems based on their surface features, rather than attending to problem semantics.

Closing Comments

Use of standard readability formulas is questionable for most types of text and is especially debatable for mathematical text, for the reasons delineated in this article. However, readability remains a vitally important consideration in constructing and selecting word problems and assessing student problem-solving performance. Evaluation of word-problem readability should include multiple factors, such as vocabulary, wording, and story concepts presented. Mestre (1988) reminds us that "language proficiency mediates cognitive functioning" (p. 215). He categorizes the types of language proficiencies that can influence problem solving as follows: proficiency with language in general, in the technical language of the domain, with the syntax and usage of language in the domain, and with the symbolic language of the domain.

Research is needed to determine more clearly which factors influence students' ability to comprehend word problems and to what degree, how we might construct more effective word problems, and best practices for teaching students to solve word problems meaningfully. These investigations should also look at potentially different responses by various student subgroups, such as those formed by gender, race/ethnicity, socioeconomic background, community type, age, and academic ability.

References

Agnihotri, R. K., & Khanna, A. L. (1992). Evaluating the readability of school textbooks: An Indian study. *Journal of Reading*, *35*, 282-288.

Allen, JoBeth. (1985, April). *Factors influencing the readability of student-generated texts* [CD-ROM]. Paper presented at the Annual Meeting of the American Educational Research Association, Chicago, IL. Available: SilverPlatter File: ERIC Item: 257 038

Anderson, Gary, Higgins, Diana, & Wurster, Stanley R. (1985). Differences in the free-reading books selected by high, average, and low achievers. *The Reading Teacher*, *39*, 326-330.

Anderson, Richard C., & Davison, Alice. (1986). *Conceptual and empirical bases of readability formulas* [CD-ROM]. Available: SilverPlatter File: ERIC Item: 281 180

Armbruster, Bonnie B. (1986). Schema theory and the design of content-area textbooks. *Educational Psychologist*, *21*, 253-267.

Beck, Isabel L., McKeown, Margaret G., Omanson, Richard C., & Pople, Martha T. (1984). Improving the comprehensibility of stories: The effects of revisions that improve coherence. *Reading Research Quarterly*, *19*, 263-277.

Bilsky, Linda Hickson, Blachman, Sheila, Chi, Chris, Chan-Mui, Athena, & Winter, Phoebe. (1986). Comprehension strategies in math problem and story contexts. *Cognition and Instruction*, *3*, 109-126.

Boaler, Jo. (1994). When do girls prefer football to fashion? An analysis of female underachievement in relation to 'realistic' mathematic contexts. *British Educational Research Journal*, *20*, 551-564.

Brown, John Seely, Collins, Allan, & Duguid, Paul. (1989). Situated cognition and the culture of learning. *Educational Researcher*, *18*(1), 32-42.

Chipman, Susan F. (1988, April). *Word problems: Where test bias creeps in* [CD-ROM]. Paper presented at the Annual Meeting of the American Educational Research Association, New Orleans, LA. Available: SilverPlatter File: ERIC Item: 300 415

Cobb, Paul. (1986). Contexts, goals, beliefs, and learning mathematics. *For the Learning of Mathematics*, *6*(2), 2-9.

Cocking, Rodney R., & Mestre, Jose P. (Eds.). (1988). *Linguistic and cultural influences on learning mathematics*. Hillsdale, NJ: Lawrence Erlbaum.

Davis-Dorsey, Judy, Ross, Steven M., & Morrison, Gary R. (1991). The role of rewording and context personalization in the solving of mathematical word problems. *Journal of Educational Psychology*, *83*, 61-68.

De Corte, Erik, Verschaffel, Lieven, & De Win, Luc. (1985). Influence of rewording verbal problems on children's problem representations and solutions. *Journal of Educational Psychology*, *77*, 460-470.

Galda, Lee. (1990). A longitudinal study of the spectator stance as a function of age and genre. *Research in the Teaching of English*, *24*, 261-278.

Garofalo, Joe. (1992). Number-consideration strategies students use to solve word problems. *Focus on Learning Problems in Mathematics*, *14*(2), 37-50.

Hembree, Ray. (1992). Experiments and relational studies in problem solving: A meta-analysis. *Journal for Research in Mathematics Education*, *23*, 242-273.

Khisty, Lena Licón. (1995). Making inequality: Issues of language and meanings in mathematics teaching with Hispanic students. In Walter G. Secada, Elizabeth

Fennema, & Lisa Byrd Adajian (Eds.), *New directions for equity in mathematics education* (pp. 225-241). Cambridge: Cambridge University Press.

Kintsch, Walter. (1986). Learning from text. *Cognition and Instruction*, *3*, 87-108.

Kintsch, Walter, & Greeno, James G. (1985). Understanding and solving word arithmetic problems. *Psychological Review*, *92*, 109-129.

Lave, Jean. (1993). Word problems: A microcosm of theories of learning. In P. Light & G. Butterworth (Eds.), *Context and cognition: Ways of learning and knowing* (pp. 74-92). Hillsdale, NJ: Erlbaum.

MacGregor, Mollie. (1990). Reading and writing in mathematics. In Jennie Bickmore-Brand (Ed.), *Language in mathematics* (pp. 100-108). Portsmouth, NH: Heinemann.

MacGregor, Mollie. (1993). Teaching mathematics in English to students of non-English-speaking background. *Multicultural Teaching to Combat Racism in School and Community*, *11*(3), 31-34.

MacGregor, Mollie, & Price, Elizabeth. (1999). An exploration of aspects of language proficiency and algebra learning. *Journal for Research in Mathematics Education*, *30*, 449-467.

Mestre, Jose P. (1988). The role of language comprehension in mathematics and problem solving. In Rodney R. Cocking & Jose P. Mestre (Eds.), *Linguistic and cultural influences on learning mathematics* (pp. 201-220). Hillsdale, NJ: Lawrence Erlbaum.

Norton, Donna E. (1991). *Through the eyes of a child: An introduction to children's literature* (3rd ed.). New York: Macmillan.

Paul, Douglas J., Nibbelink, William H., & Hoover, Hiram D. (1986). The effects of adjusting readability on the difficulty of mathematics story problems. *Journal for Research in Mathematics Education*, *17*, 163-171.

Rastall, Paul. (1994). The prepositional flux. *International Review of Applied Linguistics in Language Teaching*, *32*, 229-231.

Renninger, K. Ann. (1992). Individual interest and development: Implications for theory and practice. In K. Ann Renninger, Suzanne Hidi, & Andreas Krapp (Eds.), *The role of interest in learning and development* (pp. 361-395). Hillsdale, NJ: Erlbaum.

Rye, James. (1985). Computing readability. *Reading*, *19*(2), 110-116.

Salomon, Gavrie. (1983). The differential investment of mental effort in learning from different sources. *Educational Psychologist*, *18*, 42-50.

Shelby, Annette N. (1992). Readability formulas: One more time. *Management Communication Quarterly*, *5*, 485-495.

Shuard, Hilary, & Rothery, Andrew. (Eds.). (1984). *Children reading mathematics*. London: John Murray.

Silver, Edward A., Shapiro, Lora J., & Deutsch, Adam. (1993). Sense making and the solution of division problems involving remainders: An examination of middle school students' solution processes and their interpretations of solutions. *Journal for Research in Mathematics Education*, *24*, 117-135.

Smith, Frank. (1994). *Understanding reading: A psycholinguistic analysis of reading and learning to read*. Hillsdale, NJ: Lawrence Erlbaum Associates.

Sowder, Larry. (1989). Choosing operations in solving routine story problems. In Randall I. Charles & Edward A. Silver (Eds.). *The teaching and assessing of* *mathematical problem solving* (pp. 148-158). Reston, VA: The National Council of Teachers of Mathematics.

Theule-Lubienski, Sarah, Burgis, Kathy, & Keiser, Jane M. (1995). *What interests students in the middles grades?* Unpublished manuscript.

Thomas, Jan. (1997). Teaching mathematics in a multicultural classroom: Lessons from Australia. In Janet Trentacosta & Margaret J. Kenney (Eds.), *Multicultural and gender equity in the mathematics classroom: The gift of diversity* (34-45). Reston, VA: The National Council of Teachers of Mathematics.

Wade, Suzanne E., Schraw, Gregory, Buxton, William M., & Hayes, Michael T. (1993). Seduction of the strategic reader: Effects of interest on strategies and recall. *Reading Research Quarterly*, *28*, 93-114.

Wiest, Lynda R. (1996/1997). *The role of fantasy and real-world problem contexts in fourth- and sixth-grade students' mathematical problem solving* (Doctoral dissertation, Indiana University, 1996). *Dissertation Abstracts International*, *57*, 5091A.

Zevenbergen, Robyn. (2000). “Cracking the code” of mathematics classrooms: School success as a function of linguistic, social, and cultural background. In Jo Boaler (Ed.), *Multiple perspectives on mathematics teaching and learning* (pp. 201-223). Westport, CT: Ablex.