MISCONCEIVING OR MISNAMING?: SOME IMPLICATIONS OF TODDLERS' SYMBOLIZING FOR MATHEMATICS EDUCATION[1]

Helene Alpert Furani

New York, NY

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Children's errors in the mathematics classroom are frequently attributed to their holding "misconceptions" – faulty or incomplete understandings. This paper explores the role that language may play in students' developing understandings. It draws upon a study of toddlers' symbolizing, giving particular attention to the issue of "misnaming," occasions in which standard terms are given non-standard meanings. Processes of naming and misnaming that involve logic and rules are discussed and implications drawn to classroom mathematics. Greater attention to the role language plays in children's conceptualizations is advocated, along with a classroom discourse that encourages children to name their ideas.

Research on student conceptions and misconceptions has formed a mainstay in mathematics education research for two decades or more. Smith, diSessa & Roschelle (1993) credit this work with "significantly advanc[ing] our understanding of learning by producing detailed characterizations of the understandings students bring to instruction…" (p. 116). However, they also criticize such research as failing to acknowledge continuities between the concepts held by novices and experts and to align with constructivist views of learning, which maintain that expert knowledge builds upon rather than replaces earlier, incomplete forms. This paper accepts this stance and introduces an additional issue for consideration by educators aiming to understand novice conceptions: language. Piaget (1962) maintains:

> [L]anguage makes possible the construction of concepts, for the relationship is naturally reciprocal and the capacity for constructing conceptual representations is one of the conditions necessary for the acquisition of language (p. 221).

Hence, to gain insights into the concepts people hold, one must consider the language they use; words and ideas are inextricably intertwined. Whereas *communication* has become commonly recognized as an important component for mathematics learning (NCTM 1989, 2000), particular uses of language -- the actual words students use or could use -- has received little attention in discussions of classroom practice. On the other hand, it has been argued that the words used in mathematics are metaphorical and connect to discourses both outside and within mathematics (Lakoff & Nunez, 2000; Walkerdine,

[1] This article draws upon my dissertation study, *Toddlers' Symbolizing and its Mathematical Potential*, completed under the direction of Dr. David Pimm at Michigan State University, 2000.

1988; Pimm 1988). Educators thereby need to better understand what students take words to mean and mean by the words they say in the mathematics classroom. Lakoff and Nunez (2000) offer a detailed analysis of the "conceptual metaphors"[2] underlying many pieces of expert mathematical knowledge, but this work does not yet provide insight into how conceptual metaphors for mathematics are attained by students or the transformations conceptual metaphors may undergo on their way to resembling those that comprise the mathematical canon. This is largely unexplored territory.[3]

Recognizing the metaphorical basis, the underlying meanings of words used in the mathematics classroom, and their ramifications is difficult since as Pimm (1995) puts it, "Words…are frequently so familiar to us as adults that we fail to notice them as symbols. We are so 'at home' with them that as we speak and write, 'the words don't get in the way'" (p. 5). Stepping outside of a particular language and its accompanying conceptual system, employing cross-cultural comparison of the metaphors used in mathematics instruction can help shed light on the metaphorical basis of mathematical language. Ma (1999) discusses the use of the American term of "borrowing" as opposed to the Chinese expression of "decomposing a higher value unit" in teaching subtraction by regrouping. She explains how use of the former metaphor leads teachers to give such explanations as "if you do not have enough ones, you go over to your friend here who has plenty," which misleadingly "suggests that the two digits of the minuend are *two* independent numbers rather than two parts of *one* number" (pp. 3, 4). Indeed, as teachers may themselves frequently fail to recognize the meanings the words they use convey and the ways language shapes their own conceptualizations, the relationship of words to concepts and "misconcepts" in the mathematics classroom demands serious study.

While this paper does not begin to fill the classroom research gap, it offers a starting point in the form of more basic research. It draws upon a study of toddlers' symbolizing, giving particular attention to the toddlers' naming and "misnaming" to highlight the ways that even the youngest, newly verbal children name their ideas and hence reveal their understandings through the words they choose. Thus it advocates an expansion of language use in the mathematics classroom to allow students to name and "misname" their developing concepts, so that teachers and researchers may better understand and build upon them, and reciprocally, gain an appreciation for their own uses of language and the conceptualizations they embody.

The mathematician Raymond Wilder (1968) discusses how human beings possess "symbolic initiative" that enables them to "assign symbols to stand for objects or ideas, set up relationships between them, and operate with them on a conceptual level" (p. 5). He credits much of mathematics achievement to this uniquely human capacity. What does "symbolic initiative" actually look like as a fundamental human thought process? How might insight into this process enable educators to attend to various aspects of symbolizing in the mathematics classroom? My study of toddlers' symbolizing directly

[2] Lakoff and Nunez (2000) use the term "conceptual metaphor" to emphasize metaphor as more than a literary device; it describes how people *think* about one domain in terms of another, as revealed (and constructed) by the language they use.

[3] While some studies have focused on the value of children constructing their own "symbolizations" (e.g., Forman,1993; Lehrer, Schauble, Carpenter & Penner, 2000), this work has attended to the construction of diagrams, representations, models, inscriptions, rather than names and meanings for names.

investigates the first question and addresses the second question on a theoretical, speculative basis. While the complete study explores the various aspects of "symbolic initiative" as described by Wilder above, what I call "the symbolic continuum," this paper examines the first aspect: assigning symbols (in these cases, words) to stand for objects or ideas, specifically focusing on what happens when individual word meanings, and hence concepts, conflict with common usage.[4] This is commonly known as holding a "misconception." However, I will call it, engaging in "misnaming."

METHODS

The study involves a comparative analysis of three ethnographic case studies of toddlers under age two. All three subjects were boys, only children and the sons of educators connected to my university. I knew the first two children before the start of the study. In order observed and with the periods of observation denoted by the children's ages at the time, the subjects are: Jacob, observed from age 16 to 21 months; Jeremy, observed from age 17.5 to 21 months; and George, observed from age 19.5 to 22 months. I was a participant observer of the toddlers' everyday activity in their homes and, with George, at day care as well. I refer to Jacob and George in the results section below. Also mentioned are Jacob's parents, Ann and Carl and George's mother, Lynn.

I observed each subject for roughly two hours per visit, initially once weekly and then after some sufficient "getting to know" period, twice weekly. When with the subjects, I wrote sketchy notes on a small note pad and wrote up expanded field notes once home. After an initial period of at least one month for each subject, I introduced a video camera. I switched it on when there were particular things of interest happening, but also just let it run for long periods (20 to 30 minutes).

Parents were particularly useful informants. They kept me abreast of their children's activity between visits, supplied history and interpretations of particular actions we observed together, served as a sounding board to my spontaneous and developing analyses and even conducted their own investigations/pseudo-experiments related to my questions.

Whereas I initially attempted to use clinical-style tasks with the toddlers, I quickly became discouraged by the toddlers' failure to interact with the tasks in ways I desired. Rather, they invented their own "games" with the provided materials, which offered no hope of addressing the questions I intended the tasks to explore. However, I found that I could instead introduce subtle changes into ongoing activity and as such, test developing hypotheses in the moment. I also found that a grounded theory-based analysis of my ethnographic data offered sufficient insights into the meanings behind the toddlers' actions, whether produced by them spontaneously or in response to my manipulations.

Analysis involved careful review of field notes and video tape to compile data relevant to general categories, such as *symbolizing* and *regularity*. Related pieces of data were

[4] Whereas the study discussed here involved individual case studies of toddlers, and their invented uses of language were indeed individual, this does not preclude the possibility that several students may share invented, non-standard words and/or concepts in a given mathematics classroom.

considered together, both within each case and among the cases to determine patterns, issues and interpretations. Intermediate analytical memos were drafted as a stepping stone towards final analyses. Videotape proved an invaluable tool as I was able to view episodes again and again and come to understand things that had eluded me in the moment.

The analysis involves terms from linguistics. These include "signifier," "signified," and "sign" as given by Saussure (Walkerdine, 1988). A "signifier" is a name for something (symbol, spoken word, written word, gesture, etc.) and a "signified" the thing so named (referent, object, idea, meaning, etc.). A "sign" is a signifier and a signified fused, such as the words used in everyday spoken language, which we can utter and understand without pause. The act of using signs, producing signifiers in evocation of signifieds is known as "signification." One uses signifiers to signify signifieds.

RESULTS

Here I analyze particular episodes and chains of events that occurred with the toddlers. The chosen data reflect situations in which the toddlers signified people, objects and desires in markedly unconventional ways that conflicted directly with convention. As the relationship between the toddlers' activity and classroom mathematics may not be readily apparent, such connections are drawn in the discussion section. In the meantime, I invite the reader to simply consider the data and ponder what it may reflect of these toddlers' minds.

It is not unusual for toddlers to invent their own words for things or for adults to accept their idiosyncratic, often adorable speech. For example, Jacob called *music*, "mi-mi," and George called *ball*, "ga-ga." These words were met with acceptance by me, parents, and even babysitters. However, adults have a difficult time accepting toddlers' usage of conventional words in non-standard ways. When this occurs, to adults, the toddlers are in essence *misnaming*: using an incorrect word or phrase for a given situation or attributing to it an incorrect meaning.

In these first examples, Jacob is assigning an entire phrase a non-standard meaning. I was video taping and Jacob showed interest in the camera. I lifted him so he could see through it. The camera was pointed towards Carl on the sofa and I asked Jacob if he could see Daddy. Later that day, Jacob said several times, "want to see Daddy," to ask to look through the camera even though Carl was not in the room. At one point, I contradicted him, saying he couldn't see Daddy, but he could see his basketball hoop. Jacob echoed, "Want to see bee-ball hoop." On my next visit Jacob also said, "want to see Daddy" to mean he wanted to look through the camera, although Carl was not even in the house! I again contradicted Jacob and offered that he could see Pooh instead.

I do not believe that Jacob thought he could really see Carl in the camera each time. "Want to see Daddy" successfully signified *looking through the camera* in Jacob's first use of the expression. Jacob thereupon continued to let it signify the same meaning. He used it in subsequent situations, ones in which the word for word meaning of the expression failed to apply.

Jacob used the expression “want to build house” in similar fashion. To Jacob, it meant he wanted to play with a set of large foam squares with letter and numeral cutouts. Jacob did not literally desire to build a house, although a structure could be built from the squares that bore a mild resemblance to a house. However, other structures could also be built or in fact destroyed. Taking apart a structure actually seemed to be Jacob’s preferred activity among those he signified by “want to build house.” However, when destroying, Jacob was often met with resistance from adults who were trying to build as they understood he had asked. As in the previous example, “want to build house” likely arose from Jacob using a signifier, which signified to adults a first instance of a general activity (in this case, playing with the foam squares), to signify all subsequent instances of the activity.

In both these examples, conventional meanings of Jacob’s expressions clashed with his idiosyncratic ones. Adults therefore often failed to understand him or if they succeeded in doing so, they “corrected” him, providing conventional signifiers for the signifieds he named. The next set of episodes involves an even more obvious case of Jacob misnaming, mapping signifier to signified in a non-standard, socially unaccepted way.

It was several months into my observations that Jacob began to call me by name. When he first did, he used the wrong one. Jacob was trying to tell me to come follow him. He repeated the same sentence over and over numerous times before he gave up trying to verbalize. (He eventually came over to me and tugged my shirt, so I figured out his intentions.) One set of sounds I was able to distinguish was “wee-wa.” I asked Ann if she knew what Jacob meant. She said, “That’s Lisa, my sister.” When Jacob again called me “Wee-wa,” this time in Ann’s presence, she told him, “No, that’s not Lisa, that’s Helene.” Jacob echoed, “Haween.”

On my next visit, Jacob again called me “Wee-wa.” This time, Carl corrected him. The following visit, Jacob began to use my correct name, “Haween” when talking directly to me. However, during a game of hide and seek, when he could not find me, Jacob said aloud, “Where Haween? Where Wee-wa go?” A few visits later, the video camera caught him using both names together, saying “Here, Wee-wa, Haween” while handing me a toy, even though by that time, he was otherwise exclusively calling me “Haween,” except when deliberately in a game (discussed below).

When Jacob initially attached the signifier “Wee-wa” to me, it was a reasonable choice. “Wee-wa” indicated a female, adult person, rather than a truck for instance. Jacob may have internally thought of me as “Wee-wa” for some time before articulating this. Upon meeting with resistance, he began to use a conventionally accepted sign, “Haween,” but “Wee-wa” continued to figure in his mind. In fact, for Jacob both signifiers referred to me and at times, he used both in tandem. However, Jacob predominantly used the conventional sign in social discourse and by the end of my visits, “Wee-wa” only appeared out loud in a game. Eventually, “Haween” replaced “Wee-wa” altogether as a singular signifier for me.

Jacob initiated his game around his misnaming of me on the second visit in which he called me “Wee-wa.” When I told Carl about Jacob’s misnaming, he told him, “That’s not Lisa, that’s Helene.” Jacob contradicted him, saying, “No, Wee-wa.” Both Carl and I protested, saying, “No, Helene.” Jacob again responded, “No, Wee-wa.” Jacob and I continued this exchange a few rounds with Jacob laughing at each of his turns. At one point, I had given up the struggle and Jacob had turned his attention to his lunch, but when he finished eating, Jacob tried to resume the game by again saying, “No, Wee-wa,” several times.

The “No, Wee-wa” game made reappearances on the next four visits. Then on the fifth, the following occurred (from my field notes):

> Jacob pushes the play button on the answering machine next to the stove where he was standing. A message plays. It’s Lisa. She says her name on the message. Ann asks Jacob, “Who’s that?” Jacob says, “Haween.” Ann says, “No, who was talking on the answering machine? That was Lisa.” Jacob says, “No, Haween.” Ann says, “No, that’s Lisa.” “No, Haween.” “No, Lisa.” When Ann tells Jacob, “No, Lisa” she uses the same voice as when telling him, “You’re being silly.” Then Jacob begins to tease me, calling me “Wee-wa.” I answer, “No, Helene.” We do this exchange several times.

Jacob’s turning misnamings into games -- calling me “Wee-wa” and Lisa “Haween” -- reflect that at the time of their execution, Jacob knew the “correct” signifier for each of us. However, he saw that he could tease and prompt these games by deliberately using “wrong” or unaccepted signifiers. That Jacob knew he was teasing was evident by his frequent laughter, at times forced, during the games. This interpretation is confirmed by other evidence. Once after several acts of Jacob teasing me by first handing me my pen, then pulling it away giggling, he launched the “No, Wee-wa” game, the first interaction apparently reminding him of the second. The same thing occurred following Jacob teasing his mom by clicking a pen open after she clicked it closed, doing this a number of times in succession.

While “Wee-wa” was Jacob’s initial signifier for me, and it likely remained so in his mind for quite some time, Jacob recognized that in order to be successfully understood, he needed to adopt the convention in social discourse. His idiosyncratic signifier would only be accepted by others in play. Jacob capitulated to social pressure.

George faced a similar situation in which he gave an idiosyncratic meaning to a conventional signifier. Unlike Jacob, George outwardly resisted social pressure, at least initially. The video camera captured George looking at a book containing abstract illustrations titled, *Brown Bear, Brown Bear, What Do You See?* It had illustrations of bears on both the front and back covers and in the text. The following is a description of the video record.

> Looking at the bear picture within the text, George says, “lion” in his awed voice, and turning to me says, “It’s a big lion.” I echo him, asking, “Is that a big lion?” George says, “no, no mine” and then seeming to wait for a reaction from me says again, “lion, lion.”
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> Lynn enters the room and says, “That’s not a lion. That’s a bear. Do you want to see a picture of a lion?” As she steps out, George closes the book

> and says of the back cover, “here lion, here lion.” I ask, “What’s that? Is that the bear saying goodbye?”
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> Lynn returns with a photo book of animals, open to a page of numerous specimens. She places it in front of George and says, “Show Helene where the lion is.” George points to a photo of a lion and says in a voice filled with awe, “lion.” He makes a scared face and growls, then says, “big, big lion.”
>
> George then points to a polar bear on the same page and says “big, big lion.” Lynn responds, “That’s a bear.” Lynn then tries to interest George in some other animals pictured on the page. George offers names and sounds that they make. He points to the polar bear again, this time saying, “da” looking at me. I name, “It’s a bear.”[5]
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> Lynn turns the page and asks about animals pictured there. George points to a photo of a brown bear and says, “big, big lion.” Lynn says about that animal too, “No, that’s a bear.”
>
> George closes the photo book and returns to the first one. He flips through the pages, naming the animals and stops at the bear again. He says, “whoa, big lion.” Then he closes the book and says of the front cover, “here lion, here lion.” Lynn responds, “That’s a bear.” George says, “no, no,” and picking up the book comes over to me. He shows me the picture and says, “here lion.” I respond, “Mommy said it was a bear. I agree with her. I think it’s a bear.” Walking away with the book he says, “no, mama.”

During the episode, Lynn interpreted, “Interesting how he confuses the bear and the lion, ‘cause he always made the same noise for both of them too.” Indeed he had. A growl had been George’s idiosyncratic name for both lion and bear. He growled to evoke them, to pretend to be them and to name them, as when naming images in books. This was accepted and even encouraged by his parents and other adults. At the time of the episode given here, George had apparently learned the conventional name for one of the animals (“lion”). He therefore substituted it for his idiosyncratic one (a growl) and used it in all cases in which his idiosyncratic name had applied. In other words, *growl* could now be replaced with “lion.”

However, George was not confused. He simply decided that *bears* should be called “lion” and was very serious about it. He reiterated his point by naming various images of *bear* “lion” (five distinct ones) and repeatedly bringing up the issue. In the end, he directly expressed disagreement by saying, “no.” He also tried to win me over to his view of things.

George was particularly insistent as to his naming of the abstract images of bears. It was them he continually named and renamed and for which he stated his final opinion before letting the argument rest. By contrast with the photos, he once said, “da,” and let me

[5] “Da” was an idiosyncratic way that George asked for the names of things.

name the image "bear." His body language also showed a more willing acceptance of "bear" for the photos, although he did not produce the name himself. The photos clearly showed different animals than the one for which the name "lion" met with acceptance by Lynn and me, although they were also different from each other (polar and brown bears). George likely saw them as different animals from *lion* so he could more easily accept the use of a different name. However, the abstract images were another matter.

Abstract art is intended to be interpreted by the viewer. Do the intentions of the artist really matter? The artist for the book intended to depict a bear for the text says so. But George could not read. To him, the image was a "lion," and that was what he wanted to call it. Although George was perhaps willing to accept *bear* for realistic photos, he wished to continue to interpret the illustrations as he chose.[6]

Another instance of direct substitution of a conventional signifier for a previously accepted idiosyncratic one occurred with Jacob. Jacob had used the idiosyncratic word, "momo" to signify both the signifieds, *lawnmower* and *motorcycle*. However, towards the end of my observations, Jacob was playing with his toy lawnmower and called it "motorcycle." Carl corrected him, telling him its conventional name.

Jacob had learned that one object he called "momo" was conventionally called "motorcycle" so as with George, he substituted "motorcycle" for all cases in which "momo" had applied. Jacob's naming his *lawnmower*, "motorcycle" managed to entice me. It appeared that way in my notes. A week later my notes again describe Jacob naming his lawnmower, but this time he returned to his idiosyncratic name. He had learned that in absence of the conventional name for the object, his babyish, idiosyncratic "momo" was acceptable, whereas "motorcycle" was not.

DISCUSSION

One may wonder, and rightly so, what toddlers' early language experience has to do with learning mathematics. I hope here to make the relationship clear. As toddlers are newly learning language and constructing concepts, indeed constructing language, the processes of symbolic genesis are more readily apparent than at later ages. Toddlers are playful, creative and less restricted by the "language games" of their environs than older children and certainly adults.[7] Indeed, as adults we tend to forget that we have the capacity to name and create names along with ideas. A truly constructivist approach to teaching mathematics in the classroom, which recognizes that students construct mathematical concepts, requires that teachers allow, indeed encourage students to construct mathematical language as well. What can toddlers teach us about the processes involved in naming and the dynamics around misnaming?

For one, the toddlers' "mistakes," "confusions," "misconceptions," reveal a certain logic and rule-based nature to their formation. Such is the nature of mathematical thinking as

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[6] Two weeks later, George told me the polar bear photo was called "bear." The issue of the name of the illustration never resurfaced.
[7] "Language games" is a Wittgensteinian notion that highlights the rule-bound, game-like nature of language participation. See Ernest 1998 for a discussion on how Wittgenstein's "language games" relates to mathematics.

well and surely the thinking of students in mathematics classrooms grappling with mathematical language and ideas. The ways in which the toddlers came to misname is worth exploring and relating to parallel situations in the mathematics classroom.

Jacob's expressions, "want to see Daddy" and "want to build house," are examples of metonymy -- signifieds were named indirectly. "Metonymy" literally means, "a change of names" – the name of one thing is used to *stand* in for the name of something else. A restaurant customer is named by his order, a talk show attendee by the color of her blouse, an animal by the sound it makes. Jacob's phrases involve the name of a first instance of a situation standing in for all instances. Lakoff and Johnson (1980) describe metonymy as follows:

> Metonymy...has primarily a referential function, that is, it allows us to use one entity to *stand for* another. But metonymy is not merely a referential device. It also serves the function of understanding. For example, in the case of metonymy THE PART FOR THE WHOLE [traditionally called *synecdoche*] there are many parts that can stand for the whole. Which part we pick out determines which aspect of the whole we are focusing on (p. 36).

Hence, Jacob's use of a metonymy of first instance for all instances reveals his attentions and conceptualizations. He used a process and a rule (FIRST INSTANCE FOR ALL INSTANCES) that may have served him quite well in many situations. For example, he may have successfully learned many new words quite quickly in this manner, paying close attention to the first time he heard a name for a new object, place, activity, etc., and using it immediately to name the new referent, including in all subsequent situations; he appeared to have a rather extensive vocabulary for his age.[8] This same logic pervaded the way Jacob played hide and seek, in that he would always first search the hider's most recent hiding place before trying other possibilities. Of course, there are pitfalls to Jacob's process as it lead him to occasionally over-generalize, to over-use names when not necessarily appropriate, as with these phrases, and to perhaps fail to uncover an alternative logic to a game of hide-and-seek (such as when a player alternates among hiding places in a sequential way).

Educators need be aware of a tendency among students to over-generalize and to base signs (signifier to signified mapping) on first, necessarily limited encounters rather than broader exposure. Orr (1987) describes students who continually incorporate referents and solution processes from the first problems where certain signifiers appear in all subsequent problems where those signifiers reappear, even though the original referents are irrelevant to the new problems (e.g., x now stands for something else). In a study on middle school students' views of differences among quadrilaterals, Monaghan (2000) reports that "[s]tudents tend to overgeneralize the properties of one type of rectangle to the whole class." He sees this as a rigidity brought upon by teaching materials "in which there is a standard one-to-one object-word match" (p. 187). In other words, students

[8] If this was indeed occurring, then Jacob was quite successful at identifying which referents the new words or expressions named, as evident by his few, notable "mistakes."

likely mapped a concept onto the name "rectangle" based on simple, early encounters, a concept that became rigid and resistant to revision later on.

However, Orr and Monaghan's examples differ from Jacob's namings in that use of the same signifier (e.g., "*x*" or "rectangle") led students to think that the signifieds were also the same (e.g., "*x*" is always the unknown number of cookies and "rectangle" always looks like "a door turned sideways.") In other words, the signifiers were fused to particular meanings; they became signs in a limited and rigid way. With Jacob, the process was essentially reversed. He used signifiers in a more expansive way -- they signified a greater range of meanings than the common signs do in the wider language games in which he was participating. In Jacob's world, the adults were being more limited and rigid by comparison.

Regardless of its possibility of leading to "error" as in the examples given above, the naming process of metonymy has a wide and varied logic that is quite useful to mathematics. Lakoff and Nunez (2000) credit metonymy with enabling algebraic thinking. They explain how the metonymy of Role-for-Individual such as calling the individual who brings the pizza, "Pizza Delivery Boy," regardless of who in particular he may be, paves the way for letting a symbol such as *x* stand for any number. Metonymy also makes its way into mathematical terminology. For example, "quadrilateral" is named metonymically for its characteristic of having four sides and "triangle" is named for its characteristic of having three angles. However, the metonymy of these names is likely unapparent to students and perhaps educators as well, and as such fails to serve its function of highlighting certain characteristics and assisting understanding. Understanding may be more forthcoming if students were allowed to metonymically name mathematical objects and ideas based on the characteristics they noticed as did the toddlers studied here.[9] For example, *triangles* could be called "three-siders." Students' conceptualizations would thereby be made more apparent to educators since students would be given the chance to "call it as they see it."

The misnamings of "Wee-wa/Haween" and "Lion/Bear" reveal other considerations, particularly the formation of categories, or in other words, concepts. I doubt that Jacob just randomly chose to call me, "Wee-wa." He had other names at his disposal, such as "Mommy," "Daddy," "Granka" (for *Grandpa*) and "Barney." However, he chose to give me a name he used for another prominent female in his life about his mother's age. This is not to say that Jacob thought that Lisa and I were the *same* person, we were just named the same.

Just as Jacob may have become aware that there are many people called "mommy" in the world, he could have concluded that there are several people called "Wee-wa," who all form a particular category. The aural similarity of the two names (especially in Jacob's articulation of them) could have also contributed to his *hearing* them the same and placing both individuals in the same category. "Haween" and "Wee-wa" both share the

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[9] The toddlers used metonymy as a means for generating some of the idiosyncratic names mentioned as well. George let a *growl* stand for both a bear and a lion, naming by way of sound the animals make. Jacob let a *part* of a conventional word repeated name the object in the case of "momo" for both lawn*mo*wer and *mo*torcycle.

sound “wee.” The sounds “ha” and “wa” are quite close. The two words can be seen as inversions of each other, like mirror images, “Wee-wa,” “Haween.” For Jacob, the syllables themselves could have been the salient parts of the name with order irrelevant. The names “Wee-wa” and “Haween” thus are the *same* in all ways that matter in accord with Jacob’s possible rule. For a time, he articulated the syllables in a particular order, and then under social pressure, reversed it.

“Lion/Bear” may also reflect an instance of category formation. Lions and bears are both large hulking animals that are scary and growl. George’s naming them the same may reflect his viewing them as holding common characteristics. This does not mean he necessarily saw them as the same animal. Brown bears and polar bears appear quite different too and yet are both called “bears.” Indeed, George pretended to act afraid of the lion picture on several occasions (imagining it to be a real lion) and never did so with any of the bear images. George even revealed some of this drama when he encountered the lion photo in the episode described above, but again, not for the bear images.

Calling different things by the same name is crucial for language, since for example, not every instance of *chair* has a unique name, and for mathematics as well. Naming different things the same is part and parcel of abstracting general categories and concepts from unique experiences. The mathematician, Henri Poincaré (1982) views this ability as central to important advances in mathematics:

> Perhaps I have already said somewhere that mathematics is the art of giving the same name to different things. It is proper that these things differing in matter, be alike in form, that they may, so to speak, run in the same mold. When the language has been well chosen, we are astonished to see that all the proofs made for a certain object apply immediately to many new objects; there is nothing to change, not even the words, since the names have become the same (p. 375).

In addition to the generation of a non-standard concept or “misconcept,” “Lion/Bear” seems to reflect George’s use of certain “mathematical” rules. George originally used metonymy to idiosyncratically give the same name to both *lion* and *bear*. He growled, letting the sound the animal makes stand for its name. Upon learning a conventional name for one of the animals, namely “lion,” he directly substituted it for his idiosyncratic one, in all cases where the original name had applied. Use of mathematical notation might make this process clearer. Let a be a lion, b be a bear, c be a growl, d be the word “lion,” “<” indicate “named by,” and “=” indicate identity. Let $a < c$ and $b < c$ (the original naming). If $c = d$ (the two names mean the same) then $a < d$ and $b < d$ (direct substitution, giving the new name in place of the old). It thereby made sense “mathematically” for *bear* to be called “lion.” Jacob also appeared to use direct substitution of a conventional name for an idiosyncratic one when he replaced “momo” with “motorcycle.” However, unlike George, Jacob did not resist correction.

Educators need be aware that mathematics students could make similar direct substitutions of new for old when encountering new terminology or even new algorithms or problem solving routines, substitutions that, as with these examples, lead to apparent over-generalizations. Rather than just view them as mistakes that need correcting, one could uncover, honor and even capitalize on their logic, a logic clearly exploited by mathematics.

Still another way to view the "Haween/Wee-wa" and "Lion/Bear" situations is to understand the toddlers as making use of metaphor, noticing similarities among signifieds on an *experiential* level and by way of metaphor, naming both the same. Helene resembles "Wee-wa" so shall be called "Wee-wa." Bears resemble lions so shall be called "lions."

Experience, resemblance on a material level, forms the basis of metaphorical naming, unlike with metonymy. In metonymical namings, the talk show audience member does not resemble her red blouse, nor the diner customer his fried eggs and toast. Jacob doesn't really see daddy in the camera, he merely identifies his activity with a name given to a first instance. His subsequent namings are not based on material experience. However, experienced resemblance is essential to metaphorical namings.

Lakoff and Nunez (2000) argue that mathematics is strongly based on this essential human capacity to name by way of metaphor and indeed think by way of metaphor. They describe four "grounding metaphors" (grounded in experience) upon which arithmetic is based, which more advanced mathematics draws upon by way of "linking metaphors." Walkerdine (1988) discusses how mathematical notation may be metaphor-free, yet as soon as that notation is read, metaphor appears. She offers the example:

> [W]e could articulate [2+3=5] as: 'two plus three equals five' or 'two add three makes five' or some other combination of these or other terms. The metaphorical implications of makes and equals, for example, are quite different and certainly allow the speaker/hearer (implicitly) to link mathematical with other discourses...(p. 184).

The link of mathematics to discourses outside mathematics resembles Lakoff and Nunez's "grounding metaphors," whereas "linking metaphors" involve the relating of new pieces of mathematics to older discourses within mathematics itself. Pimm (1988) discusses how this latter form of metaphorical naming stresses particular aspects of a new concept. He offers the example of "spherical triangle," which is not really a "triangle" according to the standard definition of the term in Euclidean geometry. However, using the word "triangle" offers a metaphor that emphasizes commonalties that spherical triangles share with triangles, helping people see them as "three segments of great circles meeting pairwise in three points" (p. 33).

Whereas the human capacity to name and think by way of metaphor, to make connections to prior discourses both outside and within mathematics is of central importance to mathematics, as with the other naming processes discussed thus far, it too can lead to "misnamings." Montis (2000) discusses the case of Kay, a girl with "fuzzy phonological perception":

> Only after using Cuisenaire rods during several sessions did I comprehend that Kay was not using the standard fraction-naming scheme of numerator-denominator-*ths* and realize that Kay's calling 8/8 'eight over eight' was

> significant…Kay had constructed the numerator and denominator as separate entities with no connecting relationship…Once the *th*-language denoting the relationship of numerator to denominator was in place, Kay quickly learned to recognize and name equivalent fractions (p. 551).

Kay was in essence carrying over the meanings she had for the counting numbers such as “eight” from prior discourses. For her, the word was the same, hence the meaning was the same. Only once the word was changed could Kay’s conceptualization of the situation change. Recognizing that Kay was linking a new discourse to a prior one in a non-standard way was crucial to remedying the situation. In fact, the solution was to introduce Kay to the correct terminology.

While using clear, precise mathematical language is clearly important and potentially quite helpful to understanding, as in Kay’s case, it does not prevent all misnamings – does not preempt all links to prior discourses in non-standard ways. Indeed, I define “misnaming” the attribution of non-standard meanings to standard terms, using them in non-standard ways. An example of linking a new mathematical discourse to a prior one in such a way that misnaming occurred comes from a recent encounter I had with an eight-year-old, Arabic-speaking boy, Hilal.

I was helping Hilal with his fractions homework. He was to compare the size of fractions such as 1/3 and 1/5, written in this standard notation. He indicated 1/5 was greater than 1/3 because 5 is greater than 3; I knew I had to begin with some basic ideas. Using a square to represent a sheet cake, I asked him to cut it to share among three places – his family, his school and his grandmother’s house. (He had just successfully cut the “cake” into two and called each section “one half.” We had also spoken of the need to be fair, how his mother would be insulted if he left his family a smaller piece.) Hilal proceeded to “cut” three triangular sections from the “cake,” leaving most of it intact. I complained to him about the unused cake and asked him to use all if it. He tried again, making larger triangles, but still failed to include much of the cake. He continued to try a couple more times, still drawing triangles. I asked him to use straight cuts as he had done before with “half.” This time he did so and used all the cake, but his cuts were at an angle and seemed to reflect an attempt to still make *triangles*.

In Arabic, the word for “third” is *thulth* and for “triangle,” *muthallath*, both of course resemble the word for “three,” *thalath*.” It appears that Hilal in trying to make *thirds*, understood he was to make *triangles*, the resemblance between the Arabic words for “third” and “triangle” seeming to trigger this association. He linked the new fraction discourse to a prior discourse on shapes through a phonemic resemblance between the terms. This linking of names lead him to link ideas as well.

One could try to search for the “causes” of Hilal’s “confusion” in his instruction. Perhaps standard notation and terminology for fractions were introduced too soon. Perhaps Hilal should have had prior experiences with cutting and sharing illustrations of objects as I tried with the “cake.” Perhaps he had not yet had enough opportunities to map notation onto illustrations of fractions before moving on to more advanced exercises. Regardless, I believe the point is not to avoid all confusion, since confusion is always a part of the learning process, but rather to be aware of the role that language can play in students’ understandings and to include this consideration when assessing students' knowledge and making instructional decisions.

Why did Hilal misname in this particular way? What conception was he forming? How might considering issues around language inform the instruction of his class? What might happen if the class discussed Hilal's idea and took note of the reasoning behind it? Simply bulldozing children's errors can cause them to fester and turn sour. As in Haween/Wee-wa, children's invented namings and concepts may remain for some time after public adoption of canonical discourse. In contrast, recognizing students' namings and misnamings and honoring the reasoning behind them can help students identify the logic in their thinking, give them pride and "mathematical power." Discussions about names can also help students understand the relationships among words and ideas, the ways meanings can differ in mathematical as opposed to everyday discourse, and the importance of precision in mathematical language.

Allowing, indeed encouraging students to metaphorically name in mathematics class can help enable them to arrive at desired understandings. Monaghan (2000) discusses how when describing differences among quadrilaterals, some students used a "pulling" metaphor to talk about how parallelograms differed from rectangles. Monaghan viewed the students' perhaps spontaneous, surely non-standard (in mathematical discourse) use of this metaphor as indicating positive movement toward developing mathematical understandings of shapes:

> Whatever the source, students who are able to visualise shapes in this way have a very useful skill for engaging in higher levels of transformations and are less likely to have difficulty in seeing shapes not as immutable, fixed visual entities but as fluid (p. 191).

When students are allowed to "call it as they see it," they can arrive at powerful metaphors for themselves and others in their classrooms. Subsequently, teachers have an opportunity to understand how students "see it" and can better provide collective experiences that support the refinement and development of mathematical concepts.

CONCLUSION

This paper has drawn upon a study of toddlers' symbolizing in order to explore the important role of language in the development of concepts and apparent "misconcepts." Central to the discussion has been the notion of "misnaming," occasions in which non-standard meanings are attributed to terms present in the discourse. In actuality, misnaming is essentially one and the same as misconceiving, since a misconception occurs when one holds a non-standard concept in relation to a term already used to signify a different, broader, or more narrow meaning. George can be seen as "confused," as holding a "misconception" about what a lion is. Similarly, Hilal can be seen as holding a "misconception" about thirds. However, merely calling their understandings "misconceptions" ignores, in fact masks the role that language has played in their formation. In contrast, calling them "misnamings" can perhaps help draw attention to the role of language and assist in uncovering the logic, the rules and processes involved.

A number of such processes have been discussed here: various forms of metonymy, category formation, direct substitution of new terms for old, and metaphor. However, this list does not exhaust the possible processes for name and misname generation, nor have these processes been fully explored in this brief paper. Further research is needed into basic human thought processes upon which mathematics draws, including those involved in naming and symbolizing more broadly. In order to understand the relationship between these thought processes and classroom mathematics learning, classroom based research is also needed that investigates student-initiated language creation and the role of language in generating understandings. I also advocate that research focusing on concepts and misconceptions more fully consider the role of language.

This paper additionally contains some implicit recommendations for classroom practice. It recommends that teachers consider the role of language in their students' and indeed their own concepts and misconceptions, that classroom discussions attend to the relationships among words and ideas, and that students be encouraged to freely express their understandings, including inventing their own ways of talking about things. Following the latter suggestion can assist teachers in understanding ways their students are thinking as well as support students' developing conceptualizations. Skemp (1987) seems to advocate as much when he says,

> We should use transitional, informal notations as bridges to the formal, highly condensed notations of established mathematics. By allowing children to express thoughts in their own ways to begin with, we are using symbols already well-attached to their conceptual structure. (Skemp, 1987, p. 188).

When students express their thoughts in their own ways, in any given classroom, they will produce a multiplicity of ways, a multiplicity of names. If shared, such multiplicity can enrich understanding. Calling different things the same is important for language and mathematics, yet calling the same things differently can also produce richer, more complex understandings.

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